

Resonant Gravitational Wave Amplification - Axion and Inflaton

BRUCE A. BASSETT

International School for Advanced Studies

Via Beirut 2-4, 34014

Trieste, Italy

E-mail: bruce@stardust.sissa.it

We demonstrate the parametric amplification of the stochastic gravitational wave background during inflationary reheating and during axion/moduli oscillations. This enhances the detectability of the string/inflationary gravity wave signal, leaving a finger-print on the spectrum which might be found with future gravitational wave detectors.

1 Introduction

The nature of the cosmological background of gravitational waves is a subject of great interest at present, with many possible sources ranging from quantum fluctuations in a string-dominated phase^{1,2} or inflation³, to oscillations of cosmic string loops⁴.

Here we focus instead on a powerful mechanism for distortion and amplification of any existing gravitational wave background - namely damped parametric resonance due to oscillatory phases that the universe may have undergone. Examples are provided by reheating at the end of inflation⁵, an oscillatory dilaton phase, or during coherent axion or moduli oscillations if they form a significant portion of the dark matter.

We will discuss the amplification within the gauge-invariant Bardeen formalism. Using the gauge-invariant and covariant electric and magnetic parts of the Weyl tensor gives similar results⁶. The evolution of the transverse-traceless (TT) metric perturbations h_{ab} are naturally described by the Fourier mode functions $h_{\epsilon,k}$, where $\epsilon = \{+, \times\}$ are the polarisation states. The h_k ^a satisfy:

$$\ddot{h}_k + 3\frac{\dot{a}}{a}\dot{h}_k + \frac{k^2}{a^2}h_k = 0. \quad (1)$$

Here $a(t)$ is the scale factor of the universe which obeys the Friedmann Eq.:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\mu = \frac{\kappa}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \quad (2)$$

^aFrom now on we suppress the polarisation label. We also restrict ourselves to the case of flat spatial sections.

where $\kappa = 8\pi G$ and μ is the relativistic energy density which we have specified to be in the form of a scalar field ϕ , with potential $V(\phi)$. This gives us enough freedom to model both reheating physics and the oscillations of the axion condensate. Using Eq. (2), we can rewrite Eq. (1) as⁷:

$$\frac{d^2(a^{3/2}h_k)}{dt^2} + \left(\frac{k^2}{a^2} + \frac{3}{4}\kappa p\right)(a^{3/2}h_k) = 0, \quad (3)$$

where $p = \dot{\phi}^2/2 - V(\phi)$ is the pressure.

2 Parametric amplification of gravitational waves

Now to illustrate parametric resonance, consider the n -dimensional first order system:

$$\dot{\mathbf{y}} = P(t)\mathbf{y} \quad (4)$$

where P is any matrix with period T . Then Eq. (4) has n linearly independent normal solutions of the form:

$$\mathbf{y}_i = \mathbf{p}_i(t)e^{\mu_i t} \quad (5)$$

where the μ_i are the characteristic/Floquet exponents of the system and the \mathbf{p}_i are functions of period T . Then the n characteristic numbers defined by $\rho_i = e^{\mu_i T}$ satisfy:

$$\rho_1\rho_2\dots\rho_n = \exp\left(\int_0^T \text{Tr}P(s)ds\right) \quad (6)$$

with repeated characteristic numbers counted accordingly. The trace of $P(t)$ is thus the crucial factor determining the existence of exponentially amplified modes. If $\text{Tr}P(t) > 0$ then eq. (6) implies that:

$$\prod_{i=1}^n \rho_i > 1 \quad (7)$$

which implies that at least one of the $\rho_i > 1 \implies \mu_i > 0$ and hence by eq. (5) there is at least one unbounded, exponentially growing solution.

The archetypal example is provided by the Mathieu equation ($n = 2$):

$$\ddot{y} + [A - 2q \cos(2t)]y = 0 \quad (8)$$

Let $\mu = \max\{\mu_1, \mu_2\}$. Then μ is non-trivially related to the parameters (A, q) which span an instability chart consisting of an infinite hierarchy of resonance bands where $\mu > 0$ ⁵. Defining $\epsilon = A/(2q) - 1$, we plot μ vs. (ϵ, q) in Fig. (1)

using the piecewise quadratic approximation^{8,9}. This shows that for $\epsilon < 0$ the resonance is particularly strong.

The key point we wish to make is that Eq. (3) takes the form of the Mathieu equation (8) when $V(\phi) = m_\phi^2 \phi^2/2$, as appropriate for studying chaotic inflation⁵. In this case, ϕ evolves as $\Phi \sin(m_\phi t)$. Then the pressure is:

$$p = -\frac{m_\phi^2}{2} \Phi^2 \cos(2m_\phi t) \quad (9)$$

yielding a Mathieu Eq. with parameters:

$$A(k) = \frac{k^2}{a^2 m_\phi^2}, \quad q = \frac{3\kappa \Phi^2}{16}, \quad \epsilon = \frac{32k^2}{3\kappa a^2 m_\phi^2 \Phi^2} - 1 \quad (10)$$

showing that in this case, unlike in the case of standard reheating with a positive coupling constant⁵, $\epsilon < 0$ is possible and gravitational wave amplification can be significant if $\Phi \sim M_{pl}$.

The effect of the expansion of the universe decreases Φ and redshifts k , causing a decrease of both A and q , though ϵ remains roughly constant. The decrease of q to below unity is particularly important in stopping the resonance, and there is thus a competition between the damping effect of the expansion, and the amplification due to resonance.

One final important point regards the validity of temporal averaging. With an oscillating scalar field it is common to replace μ and p with their time-averages over an oscillation: $\bar{\mu}$ and \bar{p} . In the case that $V(\phi) \propto \phi^2$, the average equation of state is that of dust, $\bar{p} = 0$. From Eq. (3) this falsely predicts that there is no resonant amplification of the stochastic gravitational wave background during reheating or axion oscillations. This shows that temporal averaging is invalid.

3 Applications

3.1 Inflationary reheating

The gravitational wave spectrum produced during inflation is nearly scale-invariant. However, during reheating via a second order phase transition this scale-invariance is broken and the *rms* value of the spectrum is amplified. From the form of q in Eq. (10), this amplification is clearly strongly dependent on Φ , the initial amplitude of inflaton oscillations. This implies that the breaking of scale-invariance during reheating is much stronger in chaotic inflationary models than in new inflation, since Φ is much larger in the former case. More discussion can be found in⁶.

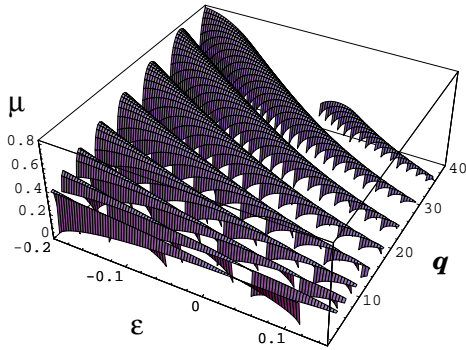


Figure 1: The Floquet index μ on the stability-instability chart for Eq. (8). Notice the rapid increase of μ for decreasing ϵ and increasing q .

3.2 The axion and massive moduli

The axion¹⁰ is an oscillating scalar field and a natural cold dark matter candidate. Unlike reheating, which lasts a very short time, the axion oscillations would last a large proportion of the universe's history, and hence might cause significant tensor amplification. The axion potential is given by^{11,12}:

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f_a}\right) \right] \quad (11)$$

with f_a the axion decay constant and $\Lambda = f_a m_a$, where m_a is the axion mass. The standard QCD axion has $\Lambda = \Lambda_{QCD} \sim 200$ MeV, $f_a \sim 10^{12}$ GeV and gains a non-zero mass due to instanton effects at an energy around Λ_{QCD} ¹². There also exist massive moduli in supergravity and superstring theories with much less constrained parameters, for example one may take $\Lambda \sim 10^{16}$ GeV and $f_a \sim M_{pl}$ ¹¹, with the moduli generically gaining mass at the epoch of supersymmetry breaking.

To understand the implications of axion oscillations, let us approximate Eq. (11) by the first, quadratic, term in the Taylor series. We can then use the results of Eq. (10) with the replacement $m_\phi^2 \rightarrow \Lambda^4/f_a^2$ so that roughly we have $A \simeq k^2 f_a^2 / (\Lambda^4 a^2)$ and $q \propto \Phi^2$. For the values given above, this yields

$$A_{QCD} \sim 10^{27} \frac{k^2}{a^2}, \quad A_{moduli} \sim 10^{-26} \frac{k^2}{a^2} \quad (12)$$

This implies that massive moduli are more likely to lead to large amplifications of the background gravity wave spectrum since $\epsilon = A/(2q) - 1 < 0$ for a huge range of modes, while in the case of the QCD axion, only a tiny fraction of the modes, near $k = 0$, have negative ϵ .

On the other hand, only if the moduli or axions started with near-Planck expectation values, $\Phi \sim M_{pl}$, will there be significant amplification in either case.

4 Conclusions

We have shown that damped parametric resonance is important in understanding gravitational wave evolution during phases where a significant component of the energy density of the universe oscillates, such as during a second order phase transition or if the dark matter lies in an oscillating scalar field.

This parametric resonance amplifies the resident stochastic background, changing the frequency dependence of the spectrum and enhancing the *rms* amplitude. This implies that the possibilities of detecting the stochastic background of gravitational waves may be better than previously thought. In addition there is the intriguing possibility of indirect detection of the axion or moduli via their finger-prints on the gravitational wave spectrum.

Acknowledgments

I thank Marco Serone for enlightening discussions.

References

1. M. Gasperini *this volume*, gr-qc/9707034 (1997)
2. R. Brustein, M. Gasperini, G. Veneziano, *Phys.Rev. D* **55**, 3882 (1997)
3. L. M. Krauss *this volume* (1997)
4. B. Allen, *this volume*; B. Allen, R. Brustein *Phys.Rev. D* **55**, 3260 (1997)
5. L. Kofman, A. Linde, and A. A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994)
6. B. A. Bassett, *Phys. Rev. D* **56**, 3439 (1997); hep-ph/9704399
7. B. A. Bassett and S. Liberati, hep-ph/9709417 (1997)
8. M. Yoshimura, *Prog. Theo. Phys.* **94**, 873 (1995)
9. H. Fujisaki, K. Kumekawa, M. Yamaguchi, and M. Yoshimura, *Phys. Rev. D* **53**, 6805 (1995)
10. R. A. Battye, E. P. S. Shellard, *Class. Quant. Grav.* **13**, A239 (1996)
11. E. W. Kolb, A. Singh and M. Srednicki, hep-ph/9709285 (1997)
12. J.E. Kim, *Physics Reports*, **150**, 1 (1987)